Radial sampling interactions in multi-dimensional sparse encoding problems using a joint decoding-reconstruction framework

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#3683

Introduction + Theory

Background:

- Many MR measures are indirectly inferred from multiple measurements. (e.g. the tag and the control image in arterial spin labelling or multi-TE images for relaxometry).
- Reconstruction and decoding of the signal of interest are often treated separately.
- Instead, treating them jointly opens up new degrees of freedom.
- Modern MRI is often undersampled and reconstructed with compressed sensing.

Theory:

- The acquisition operator, ***E***, and the point-spread-function (PSF = ***E'E***) determines the spreading of artefacts and is generally shift invariant.
- Incoherence of ***E*** is essential for compressed sensing[1] to work.
- When reconstruction and decoding of the signal are treated separately, we can only influence the incoherence of **E** through choice of trajectory.
- When considering reconstruction and decoding jointly, a more optimal combination of trajectory and encoding can be developed.

In this work:

- We used an example framework to investigate sampling and encoding interactions: golden angle radially sampled vessel-encoded arterial spin labelling (VE-ASL)[2].
- We developed a tool called the multidimensional point-spread-function (m-PSF) to explore these types of interactions.
- The m-PSF was generated by transforming (**E'E**) a delta function centred in each component and measuring its effect on every other component.

Methods

Simulated encoding-trajectory interactions:

1. Same trajectory for each encoding + 4x4 Hadamard encoding of 3 vessels and static tissue (Fig 1a – no rotation)
2. Different trajectory for each encoding (45° rotated) + 4x4 Hadamard encoding (Fig 1a)
3. Different trajectory for every other encoding with half the number of spokes per encoding compared with method 1 and 2 + paired 8x4 Hadamard encoding (Fig 1b)

Reconstruction strategies:

1. Reconstruct first – then decode (Fig 2a)
2. Decode first – then reconstruct (Fig 2b)
3. Joint reconstruction + decoding (Fig 2c)

- Compressed sensing optimisation using the Fast Iterative Soft Thresholding Algorithm (FISTA[3]) was used minimising sparsity of the image in its native space:

$$cost = \frac{1}{2}|\boldsymbol{Ex} - \boldsymbol{d}|_2^2 + \lambda|\boldsymbol{x}|_1$$

- λ = 5 for all methods

Data + simulation setup:

- 10 augmentations of a digital VE-ASL phantom were used and 10 instances of complex gaussian noise were added to each before reconstruction.
- The background component had approximately 10 times higher signal than the vessel components. Total background suppression was also tested (component 4 set to zeros).
- Images were reconstructed from a total of 16 or 64 spokes spread across the encodings.

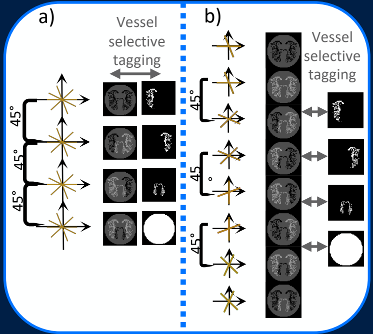

Fig 1 – Four components (3 vessels + 1 background) are encoded into four (a) or eight (b) encoded images that are acquired with a radial trajectory that is rotated for some of the encodings.

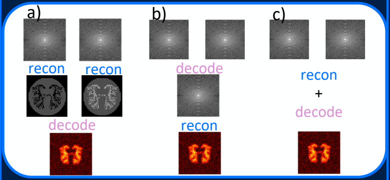

Fig 2 – To find a property of interest you can either reconstruct then decode (a), decode and then reconstruct (if same sampling points) (b), or do a joint reconstruction and decoding.

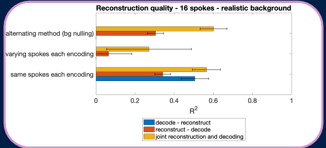

Fig 4 – Varying spokes each encoding is bad because the static tissue swamps the vessel components. The alternating spokes approach removes the background but samples more locations in k-space than using the same spokes each encoding.

Results

- All reconstructions were assessed using correlation (R^2) with ground truth data Joint reconstruction and alternating trajectory performed best (Figs 3 and 4)
- The multidimensional point spread function (m-PSF) observations (Fig 5):
  - The m-PSF is **symmetrical** (Signal from component A aliases into component B in the same way that signal from B aliases into A, etc.)
  - Component mixing determined by energy of **off-diagonal blocks**
  - Alternating trajectory nulls high energy component (background)
  - **Diagonal blocks unaffected** by which spokes are acquired in which encoding

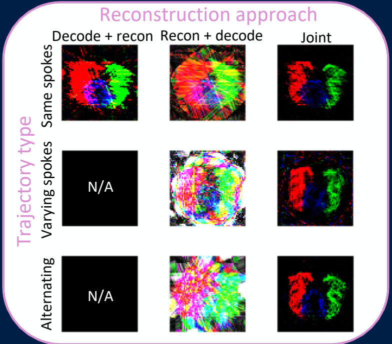

Fig 3 – Sharp and detailed reconstructions with only 16 spokes can be acquired with joint reconstruction and using the alternating spokes approach.

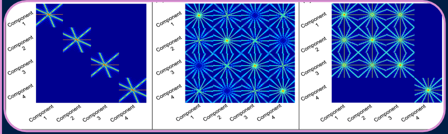

Fig 5 – Using same spokes each encoding results in no mixing of components (a). If each encoding uses a different trajectory all components mix (b). With the alternating approach one can stop one component from mixing (c). Through matching sampling and encoding we control aliasing in the component direction.

Discussion

- Joint decoding and reconstruction gives **more degrees of freedom** and produces **better results**
  - Especially when optimised for energy distribution of data
- The **m-PSF can be used as a tool** to study interactions between trajectory and other types of signal encoding
- Simple case and theory presented here. **In-vivo trials presented in abstract #1073**
- Applications beyond ASL include e.g. 4D phase imaging[4].

Watch a pre-recorded presentation here

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