The **golden ratio** is **not** the most uniform sampling strategy for **flexible temporal resolution** dynamic MRI

The SILVER method – an Improvement upon Radial Golden Ratio Sampling Within a Specified Window Size

Sophie Schauman (sophie.schauman@dtc.ox.ac.uk), Thomas W. Okell, Mark Chiew
Wellcome Centre for Integrative Neuroimaging, NDCN, University of Oxford

INTRO

- Many recent advances in dynamic MRI utilize radial sampling because of reduced sensitivity to motion and incoherent spatial aliasing when undersampled.
- Uniform sampling gives highest theoretical SNR[1], but only for a **fixed window size** (number of spokes per reconstructed frame).
- Golden ratio-based sampling methods[2,3] are often used instead, as they have near-uniform k-space coverage for **any window size.**
- Imaging experiments are mostly designed for reconstruction with a specific temporal resolution or a range of window sizes.
- *Hypothesis: When the **set or range of window sizes** are known, optimizing uniformity for these can improve image quality compared to golden ratio-based methods*
  - *-> **S**et **I**ncrement with **L**imited **V**iews **E**nhancing **R**atio **(SILVER)***

METHODS

- The set angle increment (2D) or angle and z-coordinate increment (3D) (Fig 1), was chosen by optimizing for uniformity in k-space sampling for specified set of window sizes, *S*.
- Optimization problem was formulated as:

$$\min_{(\alpha)} \max_{\Delta \in S} \frac{1}{\eta(\alpha,\Delta)} \quad \text{with } \eta(\alpha,\Delta) = \frac{U_{ref}(\Delta)}{U(\alpha,\Delta)}$$

- SNR efficiency (η) was estimated by uniformity as defined by the total electrostatic potential (U) of set of unit charges at the tip of each radial spoke. U_{ref} was defined by uniform sampling (2D), and numerically minimized potential for a window size, Δ, in 3D.
- Simulations in 2D and 3D to explore potential gain at various sets of window sizes, *S*.
- In-vivo acquisition of highly accelerated 2D arterial spin labelling angiograms[4] at three different temporal resolutions (48, 108, and 216 ms, R ≈ 19, 8.5 and 4.3 respectively) using SILVER optimised angle increments, the golden angle (≈ 111.24°), and uniform radial sampling.
- The ASL k-space data was pre-subtracted before a lightly regularised non-Cartesian SENSE reconstruction was performed using FISTA.

RESULTS

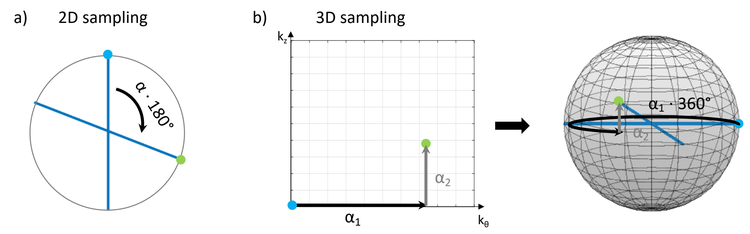
- SILVER **can** be optimized for and outperforms golden ratio based methods in both continuous window ranges and discrete window size sets (Fig 2).
- Small qualitative improvements compared to the golden angle method were observed in-vivo (Fig 3).
- SILVER has lower peak-to-sidelobe PSF than uniform sampling, but higher than golden angle sampling (Fig 4).
- Similarly, SILVER has lower noise amplification than golden angle sampling, but higher than uniform (Fig 5).

DISCUSSION

- We have presented an optimization framework for selecting more efficient increments for flexible radial sampling.
- The cost function in the optimization can easily be modified to optimize for other PSF or sampling features.
  - E.g. Temporal incoherence (important for temporally regularized reconstruction methods).
  - E.g. Maximising the mean efficiency rather than the minimum efficiency in the set of window sizes, *S*.
- For large windows, like those needed for 3D imaging, the SILVER optimization can be computationally intense.
- Once optimized, SILVER can easily be applied to any sequence that currently uses the golden ratio or golden means spoke ordering to produce efficiency gains 'for free'.

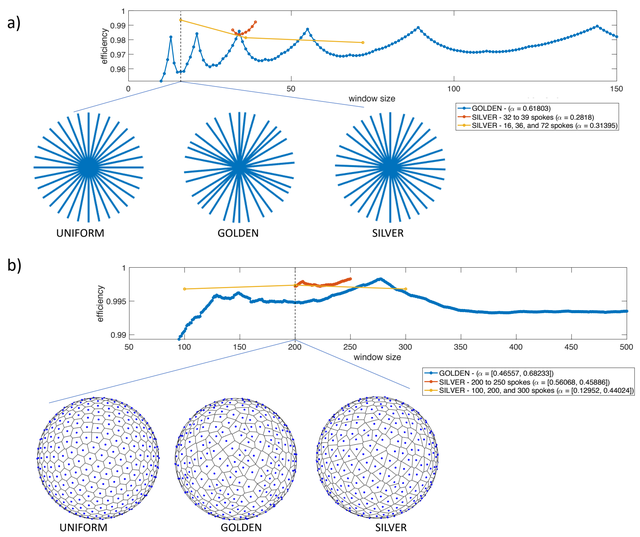
References:
1. Tsai, C M, and D G Nishimura. "Reduced Aliasing Artifacts Using Variable-Density k-Space Sampling Trajectories." Magn Reson Med. 43, no. 3 (January 1, 2000): 452–58. 2. Winkelmann S, et al., An Optimal Radial Profile Order Based on the Golden Ratio for Time-Resolved MRI. IEEE Trans. Med. Imaging 2007;26:68–76 3. Chan RW, et al., Temporal stability of adaptive 3D radial MRI using multidimensional golden means. Magn. Reson. Med. 2009;61:354–363 4. Schauman SS, Chiew M, Okell TW. Highly accelerated vessel-selective arterial spin labeling angiography using sparsity and smoothness constraints. Magn Reson Med 2020;83:892–905 doi: 10.1002/mrm.27979.

Fig 1



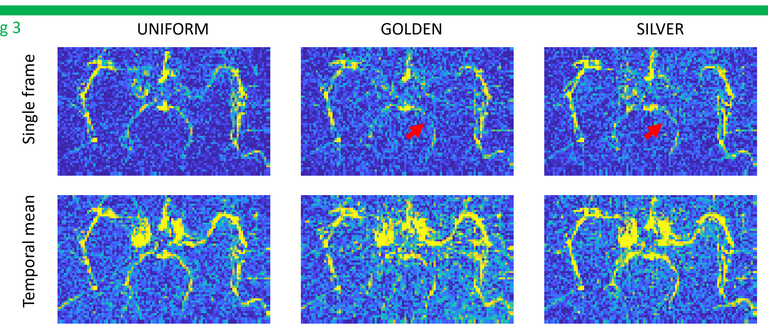
A set increment, α. Golden ratio based methods define α using the golden ratio or the 2D golden means.
a) For 2D sampling it defines the angle of each spoke relative to the previous one.
b) For 3D sampling it defines the azimuthal angle and z-coordinate of each spoke relative to the previous one.

Fig 2



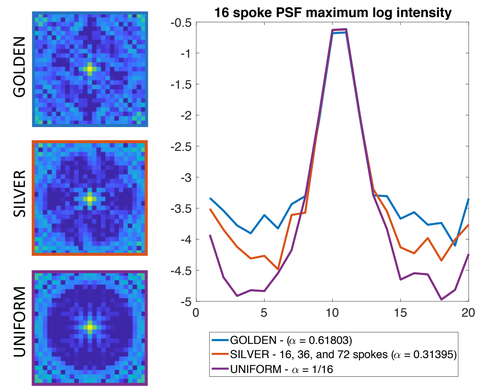
Efficiency for different set increments, α, in different example sets of window sizes.
a) shows the 2D results and a 16 spoke example trajectory, b) shows the spoke tips of 200 spokes in 3D

Fig 3



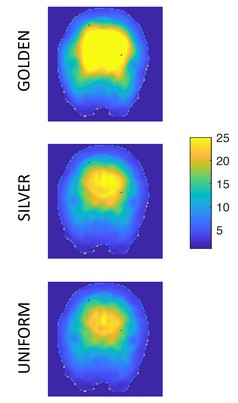
Detail of in-vivo angiograms acquired with three different trajectories (R = 4.3).
The red arrow points out an example of a vessel that is lost to noise in the golden ratio sampled image, but visible with the SILVER method.

Fig 4



PSF characteristics for a 2D 16-spoke trajectory.
The golden ratio method has the highest PSF-sidelobes, medium for SILVER, and lowest for uniform sampling.

Fig 5



g-factor maps for different trajectories (16 spokes)
High g-factors are observed in the centre of the brain for all methods due to high undersampling factors. The effect is weaker with a SILVER optimised trajectory than for the golden angle.

UNIVERSITY OF OXFORD · wellcome centre integrative neuroimaging · ROYAL ACADEMY OF ENGINEERING · EPSRC Engineering and Physical Sciences Research Council · ST CROSS COLLEGE

Take a picture to get a PDF copy